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Equations of Straight Lines

Name: ______________________________

Class: ______________________________

1. The equation of a straight line L is $kx+5y-2k=0$, where k is a constant. If L is perpendicular to the straight line $10x-4y+7=0$, find the y-intercept of L.

A. $-\frac{4}{5}$

B. $-\frac{2}{5}$

C. $\frac{2}{5}$

D. $\frac{4}{5}$

2. Find the constant k such that the straight lines $4x-6y=5k$ and $kx+9y=4$ do not intersect.

A. -9

B. -6

C. 6

D. 9

3. The straight line L is perpendicular to the straight line $8x-3y+24=0$. If the x-intercept of L is -4, then the equation of L is

A. $3x+8y+12=0$

B. $3x+8y-12=0$

C. $8x-3y+12=0$

D. $8x-3y-12=0$

4. The equation of the straight line L_1 is $3x + 4y - 24 = 0$. The straight line L_2 is perpendicular to L_1 and intersects L_1 at a point lying on the y-axis. Find the area of the region bounded by L_1, L_2 and the x-axis.

A. 24

B. $\frac{75}{2}$

C. 48

D. 75

5. Find the constant a such that the straight lines $3x + (a+2)y - 7 = 0$ and $ax - 6y + 1 = 0$ are perpendicular to each other.

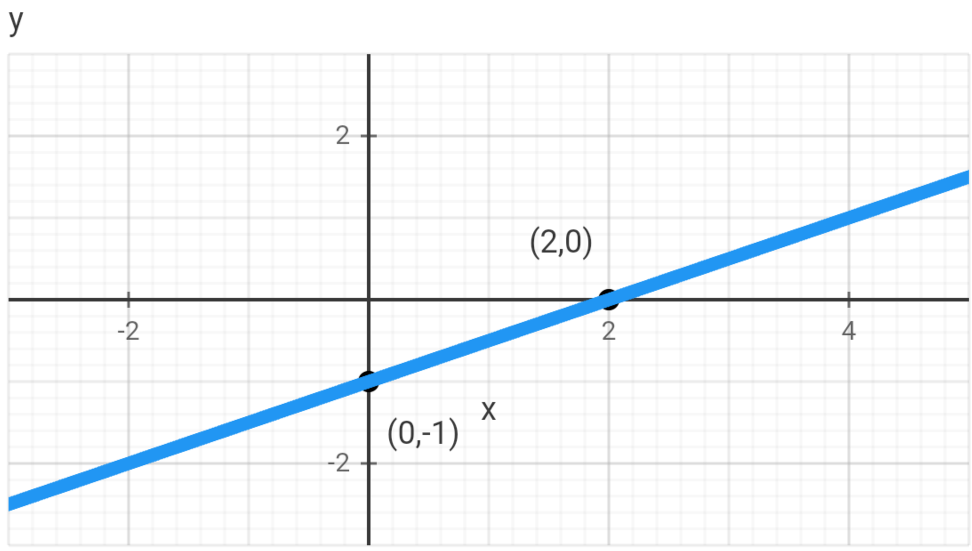
A. -6

B. -4

C. 2

D. 4

6. The figure shows the graph of the straight line $mx + ny = 4$.



Which of the following are true?
I. $m > 0$
II. $n < 0$
III. $m + n = 0$

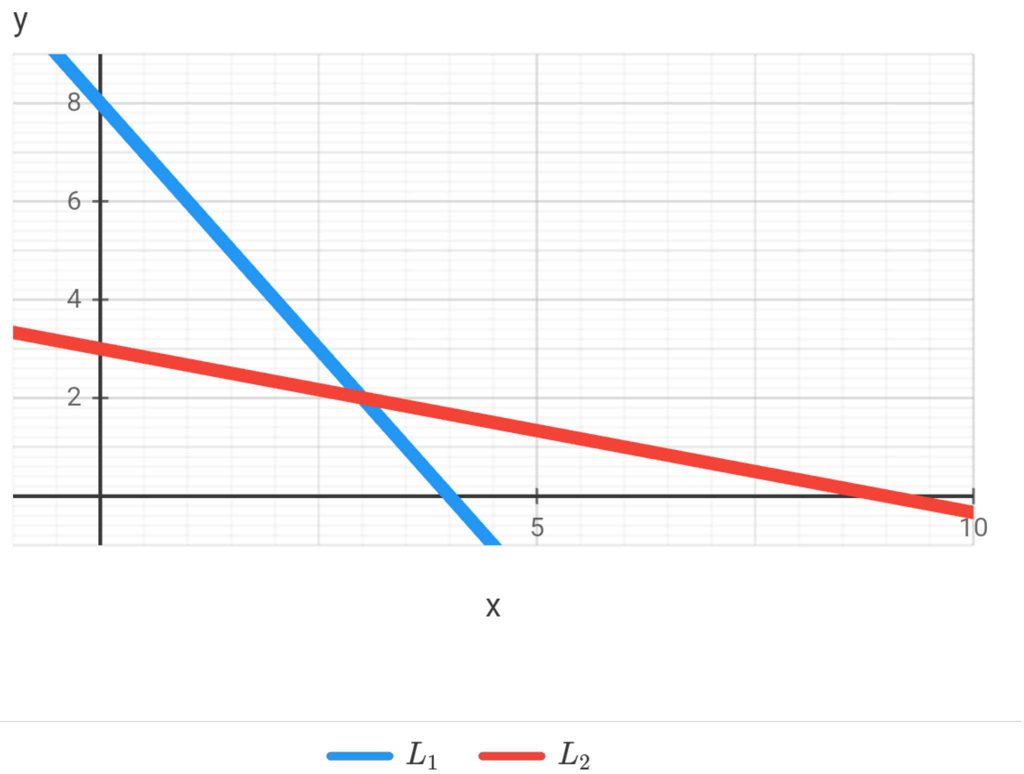
A. I and II only

B. I and III only

C. II and III only

D. I, II and III

7. In the figure, the equations of the straight lines L_1 and L_2 are $ax + y = b$ and $cx + y = d$ respectively.



Which of the following are true?
I. $a > 0$
II. $a > c$
III. $b > d$
IV. $ad > bc$

A. I, II and III only

B. I, II and IV only

C. I, III and IV only

D. I, II, III and IV

8. If the straight lines $hx + ky + 12 = 0$ and $4x + 3y - 8 = 0$ are perpendicular to each other and intersect at a point on the x-axis, then $k =$

A. -8

B. -6

C. 6

D. 8

9. The coordinates of the points A and B are $(5,-2)$ and $(-1,6)$ respectively. If C is a point lying on the straight line $x-2y=0$ such that $AC=BC$, then the x-coordinate of C is

A. -2

B. -1

C. -3

D. -4

10. The coordinates of the points A, B and C are $(2,4)$, $(6,10)$ and $(8,2)$ respectively. Let P be a point such that AP is a median of $\triangle ABC$. Find the equation of the straight line which passes through A and P.

A. $2x-5y+16=0$

B. $2x-5y-16=0$

C. $5x-2y-2=0$

D. $5x+2y-18=0$

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Answers

1 D

Rewrite $10x - 4y + 7 = 0$ in slope-intercept form:

$4y = 10x + 7$

$y = \frac{5}{2}x + \frac{7}{4}$

The slope of this line is $\frac{5}{2}$.

Since L is perpendicular to it, the slope of L is $-\frac{2}{5}$.

Rewrite $kx + 5y - 2k = 0$ as

$5y = -kx + 2k$

$y = -\frac{k}{5}x + \frac{2k}{5}$

Equating slopes: $-\frac{k}{5} = -\frac{2}{5}$, so $k = 2$.

The y-intercept of L is $\frac{2k}{5} = \frac{4}{5}$.

2 B

Two straight lines do not intersect when they are parallel and distinct.
From $4x - 6y = 5k$:

$6y = 4x - 5k$, so slope $= \frac{4}{6} = \frac{2}{3}$.

From $kx + 9y = 4$:

$9y = -kx + 4$, so slope $= -\frac{k}{9}$.

For the lines to be parallel, $-\frac{k}{9} = \frac{2}{3}$, hence $k = -6$.

When $k = -6$, the equations become $4x - 6y = -30$ and $-6x + 9y = 4$.

Multiplying the first equation by $\frac{3}{2}$ gives $6x - 9y = -45$, or $-6x + 9y = 45$, which is not the same as $-6x + 9y = 4$.
So the lines are parallel and distinct, and therefore do not intersect when $k = -6$.

3 A

From $8x - 3y + 24 = 0$:

$3y = 8x + 24$, so slope $= \frac{8}{3}$.

The slope of L is therefore $-\frac{3}{8}$.

L has x-intercept -4, so it passes through $(-4, 0)$.
Using the point-slope form:

$y - 0 = -\frac{3}{8}(x - (-4))$

$y = -\frac{3}{8}(x + 4)$

$8y = -3x - 12$

$3x + 8y + 12 = 0$

4 B

For $3x + 4y - 24 = 0$, set $x = 0$ to get $y = 6$. So L_1 meets the y-axis at $(0, 6)$.

Slope of L_1 is $-\frac{3}{4}$, so slope of L_2 is $\frac{4}{3}$.

Equation of L_2: $y - 6 = \frac{4}{3}x$, i.e. $4x - 3y + 18 = 0$.

x-intercept of L_1: set $y = 0$, $x = 8$.

x-intercept of L_2: set $y = 0$, $4x + 18 = 0$, so $x = -\frac{9}{2}$.

The bounded region is a triangle with vertices $\left(-\frac{9}{2}, 0\right)$, $(8, 0)$ and $(0, 6)$.

Area $= \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times \left(8 - \left(-\frac{9}{2}\right)\right) \times 6 = \frac{1}{2} \times \frac{25}{2} \times 6 = \frac{75}{2}$.

5 B

Slope of the first line: from $3x + (a+2)y - 7 = 0$, slope $m_1 = -\dfrac{3}{a+2}$ (provided $a \neq -2$).

Slope of the second line: from $ax - 6y + 1 = 0$, slope $m_2 = \dfrac{a}{6}$.

For perpendicular lines, $m_1 m_2 = -1$:

$$\left(-\frac{3}{a+2}\right)\left(\frac{a}{6}\right) = -1$$

$$\frac{3a}{6(a+2)} = 1$$

$$\frac{a}{2(a+2)} = 1$$

$$a = 2(a+2)$$

$$a = 2a + 4$$

$$a = -4$$

Check: $a = -4 \neq -2$, so it is valid.

6 A

The line passes through $(2, 0)$ and $(0, -1)$.

Substitute $(2, 0)$ into $mx + ny = 4$:

$2m = 4 \Rightarrow m = 2 > 0$. So I is true.

Substitute $(0, -1)$:

$-n = 4 \Rightarrow n = -4 < 0$. So II is true.

Then $m + n = 2 + (-4) = -2 \neq 0$. So III is false.

Therefore only I and II are true.

7 D

Rewrite as $y=-ax+b$ and $y=-cx+d$.
Both lines have negative slopes, so $-a<0$ and $-c<0$, giving $a>0$ and $c>0$. Thus I is true.
From the figure, L_1 is steeper than L_2. Since both slopes are negative, the slope of L_1 is smaller than the slope of L_2, i.e. $-a<-c$. Multiplying both sides by -1 gives $a>c$. Thus II is true.
The y-intercept of L_1 is greater than that of L_2, so $b>d$. III is true.
The x-intercepts are $\frac{b}{a}$ and $\frac{d}{c}$ respectively. From the figure, $\frac{b}{a}<\frac{d}{c}$. Since a, c, b and d are all positive, multiplying both sides by ac yields $bc<ad$, i.e. $ad>bc$. IV is true.
Therefore I, II, III and IV are all true.

8 D

Slope of $4x+3y-8=0$ is $-\frac{4}{3}$.
Slope of $hx+ky+12=0$ is $-\frac{h}{k}$.
Perpendicular: product of slopes $=-1$:

$$\left(-\frac{h}{k}\right)\left(-\frac{4}{3}\right)=\frac{4h}{3k}=-1$$

$$4h=-3k\Rightarrow h=-\frac{3k}{4}.$$

They meet on the x-axis, so put $y=0$:
From the second line: $4x-8=0\Rightarrow x=2$.
From the first line: $h(2)+12=0\Rightarrow 2h=-12\Rightarrow h=-6$.
Then $-6=-\frac{3k}{4}\Rightarrow 6=\frac{3k}{4}\Rightarrow k=8$.

9 A

C lies on the line $x - 2y = 0$, so let $C = (2t, t)$.

Since C is equidistant from A and B, C lies on the perpendicular bisector of AB.

Midpoint of AB: $\left(\frac{5+(-1)}{2}, \frac{-2+6}{2}\right) = (2, 2)$.

Slope of AB: $\frac{6-(-2)}{-1-5} = \frac{8}{-6} = -\frac{4}{3}$.

Slope of the perpendicular bisector: $\frac{3}{4}$.

Equation of the perpendicular bisector: $y - 2 = \frac{3}{4}(x - 2)$

$4(y-2) = 3(x-2)$

$4y - 8 = 3x - 6$

$3x - 4y + 2 = 0$.

Solve simultaneously with $x - 2y = 0$ (i.e. $x = 2y$):

$3(2y) - 4y + 2 = 0$

$6y - 4y + 2 = 0$

$2y = -2$

$y = -1$, so $x = -2$.

The x-coordinate of C is -2.

10 A

The midpoint M of BC is $\left(\frac{6+8}{2}, \frac{10+2}{2}\right) = (7, 6)$.

Since AP is a median, P lies on the line AM, so the required line is the line through A $(2, 4)$ and M $(7, 6)$.

Slope $= \frac{6-4}{7-2} = \frac{2}{5}$.

Equation: $y - 4 = \frac{2}{5}(x - 2)$

$5(y-4) = 2(x-2)$

$5y - 20 = 2x - 4$

$2x - 5y + 16 = 0$.