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Laws of Probability and Conditional Events

Name: ______________________________

Class: ______________________________

1. In a class, the set of students who like football is denoted by F and the set of students who like basketball is denoted by B. Which of the following represents the set of students who like football or basketball but not both?

A. $F \cap B$

B. $F \cup B$

C. $(F \cup B) \setminus (F \cap B)$

D. $F \setminus B$

2. Events A and B are mutually exclusive. It is given that $P(A) = 0.35$ and $P(B) = 0.28$. Find $P(A \cup B)$.

A. 0.07

B. 0.63

C. 0.098

D. 0.91

3. The probability that it rains on a certain day in March is 0.4. Find the probability that it does not rain on that day.

A. 0.24

B. 0.4

C. 0.6

D. 1.4

4. Two fair coins are tossed independently. Let A be the event that the first coin shows a head and B be the event that the second coin shows a head. Which of the following is true?

A. $P(A \cap B) = 0$

B. $P(A \cup B) = P(A) + P(B)$

C. $P(A \cap B) = P(A)P(B)$

D. $P(A|B) = 0$

5. A bag contains 5 red balls and 3 blue balls. Two balls are drawn at random one after another without replacement. Find the probability that the first ball is red and the second ball is blue.

A. $\frac{15}{56}$

B. $\frac{15}{64}$

C. $\frac{5}{14}$

D. $\frac{8}{56}$

6. It is given that $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$. Find $P(A|B)$.

A. 0.2

B. 0.3

C. 0.5

D. 0.6

7. A fair die is thrown. Let A be the event that an even number is obtained and B be the event that a number greater than 4 is obtained. Find $P(A \cup B)$.

A. $\frac{1}{2}$

B. $\frac{5}{6}$

C. $\frac{2}{3}$

D. $\frac{1}{3}$

8. When Ken shoots at a target, the probability that he hits the target is 0.8. He shoots twice independently. Find the probability that he hits the target exactly once.

A. 0.16

B. 0.32

C. 0.36

D. 0.64

9. A box contains 4 white cards and 6 black cards. Two cards are drawn at random one after another without replacement. Given that the second card is white, find the probability that the first card is also white.

A. $\frac{1}{5}$

B. $\frac{1}{3}$

C. $\frac{2}{5}$

D. $\frac{2}{9}$

10. There are three questions in a quiz. The probabilities that Amy answers the first, second and third questions correctly are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. Assume that her answers to the three questions are independent. Find the probability that she answers at most 2 questions correctly.

A. $\frac{1}{24}$

B. $\frac{5}{24}$

C. $\frac{23}{24}$

D. $\frac{11}{12}$

Answers

1 C

The set of students who like football or basketball but not both is the symmetric difference of F and B.

This consists of students in F only or in B only, which can be written as
$(F \cup B) \setminus (F \cap B)$
or equivalently $(F \setminus B) \cup (B \setminus F)$.

Among the options, $(F \cup B) \setminus (F \cap B)$ matches this meaning.

2 B

If two events are mutually exclusive, they cannot occur together, so $P(A \cap B) = 0$.

By the addition law,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.35 + 0.28 - 0$
$= 0.63.$

3 C

Let R be the event that it rains. Then the event that it does not rain is the complementary event R'.

By the property of complementary events,
$P(R') = 1 - P(R) = 1 - 0.4 = 0.6.$

4 C

Each fair coin has $P(\text{head}) = \frac{1}{2}$.

Since the tosses are independent,
$P(A \cap B) = P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

Also $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{2}$, so $P(A)P(B) = \frac{1}{4}$.

Hence $P(A \cap B) = P(A)P(B)$, which is the definition of independent events. Option C states this correctly.

5 A

Total number of balls at the start is $5 + 3 = 8$.

Probability that the first ball is red:
$P(\text{1st red}) = \frac{5}{8}$.

Given the first was red, 7 balls remain with 3 blue, so
$P(\text{2nd blue}|\text{1st red}) = \frac{3}{7}$.

By the multiplication law for dependent events,
$P(\text{1st red and 2nd blue}) = \frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$.

6 D

Conditional probability is defined by
$P(A|B) = \frac{P(A \cap B)}{P(B)}$, provided $P(B) \neq 0$.

Substitute the given values:
$P(A|B) = \frac{0.3}{0.5} = 0.6$.

7 C

Sample space: $\{1, 2, 3, 4, 5, 6\}$.

$A = \{2, 4, 6\}$, so $P(A) = \frac{3}{6} = \frac{1}{2}$.

$B = \{5, 6\}$, so $P(B) = \frac{2}{6} = \frac{1}{3}$.

$A \cap B = \{6\}$, so $P(A \cap B) = \frac{1}{6}$.

By the addition law,
$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6}$
$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$.

8 B

Let H denote a hit and M a miss. Then $P(H) = 0.8$ and $P(M) = 0.2$.

Hitting exactly once can occur in two mutually exclusive ways: HM or MH.

Since the shots are independent,
$P(\mathrm{HM}) = 0.8 \times 0.2 = 0.16$
$P(\mathrm{MH}) = 0.2 \times 0.8 = 0.16$

By the addition law,
$P(\text{exactly one hit}) = 0.16 + 0.16 = 0.32$.

9 B

We need $P(\text{1st white}|\text{2nd white})$.

Let W_1 and W_2 be the events that the first and second cards are white respectively.

By definition,
$P(W_1|W_2) = \dfrac{P(W_1 \cap W_2)}{P(W_2)}$.

$P(W_1 \cap W_2) = \dfrac{4}{10} \times \dfrac{3}{9} = \dfrac{12}{90} = \dfrac{2}{15}$.

For $P(W_2)$, condition on the first card:
$P(W_2) = P(W_1)P(W_2|W_1) + P(W_1')P(W_2|W_1')$
$= \dfrac{4}{10} \cdot \dfrac{3}{9} + \dfrac{6}{10} \cdot \dfrac{4}{9} = \dfrac{12}{90} + \dfrac{24}{90} = \dfrac{36}{90} = \dfrac{2}{5}$.

Therefore
$P(W_1|W_2) = \dfrac{\frac{2}{15}}{\frac{2}{5}} = \dfrac{2}{15} \times \dfrac{5}{2} = \dfrac{1}{3}$.

10 C

"At most 2 correctly" is the complementary event of "all 3 correctly".

Since the answers are independent,
$P(\text{all 3 correct}) = \dfrac{1}{2} \times \dfrac{1}{3} \times \dfrac{1}{4} = \dfrac{1}{24}$.

Hence
$P(\text{at most 2 correct}) = 1 - \dfrac{1}{24} = \dfrac{23}{24}$.